Using Index Matrices for Handling Multiple Scenarios in Decision Making

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ABSTRACT: In a general setting, decision makers can face the situation where different criteria apply for the different options among which they have to choose the best one. We call such options scenarios. In this paper we study criteria evaluation in a multi-criteria decision method for the handling of different scenarios. A novel intuitionistic fuzzy evaluation method based on index matrices is proposed. This approach permits to efficiently handle the indifference caused by the inapplicability of criteria in specific scenarios. An illustrative example is provided.
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1 Introduction

Decision support systems (DSS) aim to help decision makers selecting the best option from a set of 'competing' options. Multi-criteria decision methods (MCDM) are a commonly used approach to solve decision making problems [9, 4]. Underlying MCDM is the following three-step basic approach. Firstly, decision makers specify multiple criteria to express their preferences regarding different characteristics of the options under consideration. Secondly, these criteria are evaluated for each option, using the data that are valid for that option, and the resulting evaluation scores are aggregated to an overall evaluation score. Thirdly, all overall evaluation scores are compared to each other in order to select the best option in view of the preferences of the decision makers. Commonly used MCDM methods use simple additive scoring (SAS, also known as SAW, simple additive weighting), ordered weighted average (OWA), outranking methods (ELECTRE, PROMETHEE), the multi-attribute value technique (MAVT), the multi-attribute utility technique (MAUT), the analytic hierarchy process (AHP), or logic scoring of preference (LSP). A comparative study on these methods can be found in [6].

In most work on MCDM it is assumed that competing options are of the same type and described by the same characteristics. For example, if one has to decide on which car to buy the different options are the different cars under consideration and the characteristics of car can be safety, maximum speed, fuel consumption, colour, type, etc. In this paper, we study a different, more general situation where the competing options are not necessarily of the type and each option might have different characteristics. In such a case, we call the option a scenario. Consider that one has to choose on how to travel from A to B. Different scenarios might be to travel by train, car, bus, plane or boat. Depending on the scenario, different characteristics might be important. Weather predictions might be very important if you travel by plane or boat and not relevant when travelling by train. Road conditions might be only relevant if you travel by car or bus, while total travel time is usually relevant for all scenarios.

Having to explicitly deal with the inapplicability of criteria in some scenarios is challenging and to the best of our knowledge not studied in the context of MCDM. Intuitionistic fuzzy sets [1] offer an adequate mathematical framework to cope with the indifference that goes along with the handling of inapplicable criteria, whereas index matrices [3] can be used for the handling and aggregation of the evaluation scores. In the remainder of this paper, we describe how index matrices and intuitionistic fuzzy evaluation can be used for the handling multiple scenarios in MCDM.

The organization of the paper is as follows. In Section 2 we present some preliminaries on MCDM and scenario handling. In Section 3 we describe scenario modelling and evaluation using index matrices. Next, we discuss the aggregation of the evaluation scores and decision making in Section 4. In Section 5, we give an illustrative example. Finally, we state some conclusions in Section 6.

2 Preliminaries

Consider the situation where a decision maker has to find the most suitable option o^* among a set of available options $O = \{o_1, \ldots, o_n\}$. In a setting of MCDM, each option is characterized by a set of characteristics (or attributes) $\{A_1, \ldots, A_m\}$ on the values of which the decision maker can specify preferences. The preferences for attribute A_i, $i = 1, \ldots, m$ are specified by means of a criterion c_{A_i} that determines which values for A_i are preferred and which are not. Partial preferences can be specified with a fuzzy criterion. In such a case the criterion is specified by the membership function of a fuzzy set over the domain dom_{A_i} of acceptable values for A_i [8], i.e. $c_{A_i} : dom_{A_i} \to [0,1]$.

To find out how good an option o_j satisfies the preferences of the decision maker, all criteria c_{A_i}, $i = 1, \dots, m$ firstly have to be evaluated using the data of o_j. In a basic fuzzy approach, this boils down to computing the membership degree $s_{i,j} = c_{A_i}(o_j[A_i])$ of the actual value $o_j[A_i]$ of A_i for o_j. The membership degree $s_{i,j} \in [0,1]$ is called an elementary suitability degree and denotes to what extent option o_j is suitable with respect to its characteristic A_i. Secondly, all all elementary suitability degrees $s_{i,j}$, $i = 1, \dots, m$ have to be aggregated to obtain an overall suitability degree s_j for option o_j. The options o_j with highest overall suitability degree s_j are then considered to be the most suitable for the decision maker.

In traditional MCDM it is mostly considered that the same criteria c_{A_i}, $i = 1, \dots, m$ apply to all options o_j. In this paper, we study the more general case where this consideration does not hold. Hence, we explicitly consider that different subsets of criteria apply to different options. In such a general setting, we will speak about *scenarios* instead of options.

3 Scenario Modelling and Evaluation

The general MCDM case under consideration is described as follows. We consider a set of n scenarios $O = \{o_1, \dots, o_n\}$ and a set of m characteristics $A = \{A_1, \dots, A_m\}$. For each scenario o_j, $j = 1, \dots, n$, only a subset $A_{o_j} \subseteq A$ of characteristics are applicable. Moreover, for each applicable characteristic $A_i \in A_{o_j}$, a scenario specific fuzzy criterion $c^{o_j}_{A_i} : dom_{A_i} \to [0,1]$ is specified. For each scenario o_j, $j = 1, \dots, n$, all its scenario specific criteria $c^{o_j}_{A_i}$, $A_i \in A_{o_j}$, are evaluated by computing the membership degree $s_{i,j} = c^{o_j}_{A_i}(o_j[A_i])$ of the actual value $o_j[A_i]$ of the attribute A_i for o_j.

In this study, we propose to use an index matrix for properly handling all computed membership degrees. An index matrix is generally defined as follows.

Let $\mathcal{I}$ be a fixed set of indices and $\mathcal{R}$ be the set of real numbers.

Let operations $\circ, * : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ be fixed. For example, they can be the pairs, $\langle \circ, * \rangle \in \{\langle \times, + \rangle, \langle \max, \min \rangle, \langle \min, \max \rangle\}$, or others.

Let the standard sets K and L satisfy the condition: $K, L \subset \mathcal{I}$. Let the standard set-theoretical operations be defined over these sets. An object

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \dots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \dots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \dots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \dots & a_{k_m,l_n} \end{array}$$

where

$$K = \{k_1, k_2, ..., k_m\} \text{ and } L = \{l_1, l_2, ..., l_n\},$$

and

$$\forall\, 1 \le i \le m, \forall\, 1 \le j \le n : a_{k_i,l_j} \in \mathcal{R}$$

is called an *index matrix with real number elements* ($\mathcal{R}$-IM) [3].

Now, let $K = \{1, \ldots, m\}$ be the set of indices i of all the criteria $c^{o_j}_{A_i}$ and let $L = \{1, \ldots, n\}$ be the set of indices j of all scenarios o_j. Furthermore, let $a_{i,j}$ be determined by

$$a_{i,j} = \begin{cases} c^{o_j}_{A_i}(o_j[A_i]) & \text{if } A_i \in A_{o_j}, \\ \perp & \text{else.} \end{cases}$$

Hence, $a_{i,j}$ equals the suitability degree $s_{i,j}$ if characteristic A_i applies to scenario o_j. If not, the characteristic is not applicable and the criterion cannot be evaluated, which we denote by $a_{i,j} = \perp$. So, we obtain the matrix

$$E = \begin{array}{c|cccc} & 1 & 2 & \ldots & n \\ \hline 1 & a_{1,1} & a_{1,2} & \ldots & a_{1,n} \\ 2 & a_{2,1} & a_{2,2} & \ldots & a_{2,n} \\ \vdots & & & & \\ m & a_{m,1} & a_{m,2} & \ldots & a_{m,n} \end{array}$$

This matrix E is then transformed to the matrix E^i representing the intuitionistic fuzzy evaluation of each criterion, with respect to all scenarios

$$E^i = \begin{array}{c|cccc} & 1 & 2 & \ldots & n \\ \hline 1 & \langle \mu_{1,1}, \nu_{1,1} \rangle & \langle \mu_{1,2}, \nu_{1,2} \rangle & \ldots & \langle \mu_{1,n}, \nu_{1,n} \rangle \\ 2 & \langle \mu_{2,1}, \nu_{2,1} \rangle & \langle \mu_{2,2}, \nu_{2,2} \rangle & \ldots & \langle \mu_{2,n}, \nu_{2,n} \rangle \\ \vdots & & & & \\ m & \langle \mu_{m,1}, \nu_{m,1} \rangle & \langle \mu_{m,2}, \nu_{m,2} \rangle & \ldots & \langle \mu_{m,n}, \nu_{m,n} \rangle \end{array}$$

where

$$\mu_{i,j} = \begin{cases} \frac{1}{S_i} a_{i,j} & \text{if } a_{i,j} \neq \perp \text{ and } S_i \neq 0 \\ 0 & \text{else} \end{cases}$$

and

$$\nu_{i,j} = \begin{cases} \frac{1}{S_i} \sum_{k=1; k \neq j; a_{i,k} \neq \perp}^{n} a_{i,k} & \text{if } a_{i,j} \neq \perp \text{ and } S_i \neq 0 \\ 0 & \text{else} \end{cases}$$

with

$$S_i = \sum_{k=1; a_{i,k} \neq \perp}^{n} a_{i,k}.$$

Obviously, $\mu_{i,j} + \nu_{i,j} \leq 1$, $1 \leq i \leq m$, $1 \leq j \leq n$, i.e., the pair $\langle \mu_{i,j}, \nu_{i,j} \rangle$ is an intuitionistic fuzzy pair in the sense of [2]. The number

$$\pi_{i,j} = 1 - \mu_{i,j} - \nu_{i,j}$$

corresponds to the inapplicability of a characteristic/criterion so that there is no opinion about it.

4 Aggregation and Decision Making

In order to come to an overall impression of which scenario is most favourable, the intuitionistic fuzzy pairs in matrix E^i should be aggregated per scenario.

Let $k_0 \notin K$ be an index. Then the following aggregation operators can be considered.

Maximal-row-aggregation

$\rho_{max}(E^i, k_0) =$

	1	2	...	n
k_0	$\langle \max\limits_{1\le i\le m}(\mu_{i,1}), \min\limits_{1\le i\le m}(\nu_{i,1})\rangle$	$\langle \max\limits_{1\le i\le m}(\mu_{i,2}), \min\limits_{1\le i\le m}(\nu_{i,2})\rangle$	...	$\langle \max\limits_{1\le i\le m}(\mu_{i,n}), \min\limits_{1\le i\le m}(\nu_{i,n})\rangle$

Minimal-row-aggregation

$\rho_{min}(E^i, k_0) =$

	1	2	...	n
k_0	$\langle \min\limits_{1\le i\le m}(\mu_{i,1}), \max\limits_{1\le i\le m}(\nu_{i,1})\rangle$	$\langle \min\limits_{1\le i\le m}(\mu_{i,2}), \max\limits_{1\le i\le m}(\nu_{i,2})\rangle$	...	$\langle \min\limits_{1\le i\le m}(\mu_{i,n}), \max\limits_{1\le i\le m}(\nu_{i,n})\rangle$

Using the first aggregation operator ρ_{max}, we obtain *optimistic* evaluations for the evaluated scenarios, with the second operator ρ_{min} *pessimistic* evaluations are acquired.

Other aggregation strategies are possible. For example, one can opt to neglect criteria that are not applicable and apply an intuitionistic fuzzy t-norm or t-conorm on the intuitionistic fuzzy pairs that differ from $\langle 0, 0\rangle$. The aggregation then becomes

t-Norm-row-aggregation

$\rho_{\mathcal{T}}(E^i, k_0) =$

	1	2	...	n
k_0	$\mathcal{T}_{\substack{1\le i\le m \\ \langle\mu_{i,1},\nu_{i,1}\rangle\neq\langle 0,0\rangle}} \langle \mu_{i,1}, \nu_{i,1}\rangle$	$\mathcal{T}_{\substack{1\le i\le m \\ \langle\mu_{i,2},\nu_{i,2}\rangle\neq\langle 0,0\rangle}} \langle \mu_{i,2}, \nu_{i,2}\rangle$	...	$\mathcal{T}_{\substack{1\le i\le m \\ \langle\mu_{i,n},\nu_{i,n}\rangle\neq\langle 0,0\rangle}} \langle \mu_{i,n}, \nu_{i,n}\rangle$

where $\mathcal{T}$ denotes an intuitionistic fuzzy t-norm like [5]

1. $\mathcal{T}^{inf}(\langle\mu_1, \nu_1\rangle, \langle\mu_2, \nu_2\rangle) = \langle \min(\mu_1, \mu_2), \max(\nu_1, \nu_2)\rangle$,
2. $\mathcal{T}^{sup}(\langle\mu_1, \nu_1\rangle, \langle\mu_2, \nu_2\rangle) = \langle \max(\mu_1, \mu_2), \min(\nu_1, \nu_2)\rangle$,
3. $\mathcal{T}^{alg}(\langle\mu_1, \nu_1\rangle, \langle\mu_2, \nu_2\rangle) = \langle \mu_1\mu_2, \nu_1 + \nu_2 - \nu_1\nu_2\rangle$.

A ranking function can then be used to find the best scenario. Two intuitionistic fuzzy pairs $\langle\mu_1, \nu_1\rangle$ and $\langle\mu_2, \nu_2\rangle$ can for example be ranked using the following order relation $\succ$ [7], defined by:

1. If $\mu_1 - \nu_1 > \mu_2 - \nu_2$, then $\langle\mu_1, \nu_1\rangle \succ \langle\mu_2, \nu_2\rangle$.
2. If $\mu_1 - \nu_1 = \mu_2 - \nu_2$, then

(a) If $\mu_1 + \nu_1 > \mu_2 + \nu_2$, then $\langle \mu_1, \nu_1 \rangle \succ \langle \mu_2, \nu_2 \rangle$;

(b) If $\mu_1 + \nu_1 = \mu_2 + \nu_2$, then $\langle \mu_1, \nu_1 \rangle = \langle \mu_2, \nu_2 \rangle$.

After ranking the intuitionistic fuzzy pairs resulting from the aggregation of the intuitionistic fuzzy pairs of each scenario using the order relation $\succ$, we choose the scenario(s) corresponding to the highest ranked intuitionistic fuzzy pair as being the most favourable one(s).

5 Illustrative Example

Consider the situation where a person has to travel from location A to location B and has to choose between the following travel scenarios: (1) by plane, (2) by train, (3) by car, and (4) by boat. The applicable characteristics for each of these options are as follows.

		(1) *plane*	(2) *train*	(3) *car*	(4) *boat*
(1)	*duration*	x	x	x	x
(2)	*price*	x	x	x	x
(3)	*weather_prediction*	x		x	x
(4)	*road_condition*			x	

For each applicable characteristic A_i a scenario specific criterion $c_{A_i}^{o_j}$ is specified. This is for example because preferences for weather conditions might be different for travel by car and travel by boat.

Assume that criteria evaluation results in the evaluation matrix

$$E = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0.6 & 0.4 & 0.1 \\ 2 & 0.2 & 0.5 & 1 & 0.7 \\ 3 & 0.4 & \perp & 0.6 & 0.2 \\ 4 & \perp & \perp & 0.2 & \perp \end{array}$$

The transformed matrix E^i then becomes

$$E^i = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & \langle 0.48, 0.52 \rangle & \langle 0.29, 0.71 \rangle & \langle 0.19, 0.81 \rangle & \langle 0.05, 0.95 \rangle \\ 2 & \langle 0.08, 0.92 \rangle & \langle 0.21, 0.79 \rangle & \langle 0.42, 0.58 \rangle & \langle 0.29, 0.71 \rangle \\ 3 & \langle 0.33, 0.67 \rangle & \langle 0, 0 \rangle & \langle 0.50, 0.50 \rangle & \langle 0.17, 0.83 \rangle \\ 4 & \langle 0, 0 \rangle & \langle 0, 0 \rangle & \langle 1, 0 \rangle & \langle 0, 0 \rangle \end{array}$$

Applying a t-norm-row-aggregation with t-norms $\mathcal{T}^{inf}$, $\mathcal{T}^{sup}$ and $\mathcal{T}^{alg}$ then respectively results in

- $\rho_{\mathcal{T}^{inf}}(E^i, k_0) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline k_0 & \langle 0.08, 0.92 \rangle & \langle 0.21, 0.79 \rangle & \langle 0.19, 0.81 \rangle & \langle 0.05, 0.95 \rangle \end{array}$

  Herewith

  $$\langle 0.21, 0.79 \rangle \succ \langle 0.19, 0.81 \rangle \succ \langle 0.08, 0.92 \rangle \succ \langle 0.05, 0.95 \rangle$$

  So with this pessimistic aggregation, travel by train is the best option.

- $\rho_{\mathcal{T}^{sup}}(E^i, k_0) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline k_0 & \langle 0.48, 0.52\rangle & \langle 0.29, 0.71\rangle & \langle 1, 0\rangle & \langle 0.29, 0.71\rangle \end{array}$

  Herewith
  $$\langle 1, 0\rangle \succ \langle 0.48, 0.52\rangle \succ \langle 0.29, 0.71\rangle$$
  So with an optimistic aggregation, travel by car is the best option.

- $\rho_{\mathcal{T}^{alg}}(E^i, k_0) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline k_0 & \langle 0.01, 0.99\rangle & \langle 0.06, 0.94\rangle & \langle 0.04, 0.96\rangle & \langle 0.002, 0.998\rangle \end{array}$

  Herewith
  $$\langle 0.06, 0.94\rangle \succ \langle 0.04, 0.96\rangle \succ \langle 0.01, 0.99\rangle \succ \langle 0, 1\rangle$$
  So with the algebraic aggregation, travel by train again turns out to be the best option.

Which aggregation operator to choose depends on the preferences of the decision maker.

6 Conclusions

In this paper we studied a more general form of multi-criteria decision problem, where different criteria apply to different options. In such a case, we call the options scenarios. To handle the problem of selecting the best scenario among a set of scenarios, index matrices and intuitionistic fuzzy criteria evaluation are used. In the paper we proposed and illustrated a basic approach for evaluating scenarios and aggregating evaluation results having to cope with the situation that not all criteria are applicable for all scenarios.

Initial examples reveal the potential and usefulness of the approach, but more research on advanced modelling of inapplicability in criteria handling and aggregation is required and planned.

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